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Creating the multidimensional entangled coherent states of two cavity modes

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Abstract. We propose a scheme for creating the multidimensional entangled coherent states of two cavity modes in context of cavity quantum electrodynamics(QED). It is pointed out that under certain condition such superposition states can approximate pair coherent states and pair cat states with a high degree of accuracy.

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The preparation of nonclassical states in well controlled condition is the subject of an intense experimental activity. The manipulation of these states leads to a better understanding of basic quantum phenomena. Recently, there has been increasing interest in the preparation of two-mode nonclassical state in order to test quantum mechanics against local hidden theory [1] and implement quantum information processing protocols [2].

Cavity QED, with Rydberg atoms crossing superconducting cavities, offers an almost ideal system for the generation of entangled states and implementation of small scale quantum information processing [3]. In the context of cavity QED, numerous theoretical schemes for generating entangled states of many atoms and nonclassical states of cavity fields have been proposed [4], which led to experimental realization of the Einstein-Podolsky-Rosen (EPR) state [5] of two atoms, Greenberger-Horne-Zeilinger (GHZ) state [6] of three parties (two atoms plus one cavity mode), Schrödinger cat state [7] and Fock state [8] of single-mode cavity field. Most of the schemes are based on the interaction of atoms and single-mode cavity field. An experiment has been reported for preparing two modes of a superconducting cavity in a maximally entangled state by using a sequence of interactions of a atom with two cavity modes [9]. This experiment opens up a new possibility for quantum state engineering and quantum information processing using multiple modes in a superconducting cavity. In reference [10], Ikram et al. proposed a scheme for generation of Bell states between two cavity modes. In reference [11], Solano et al. proposed a scheme to generate two-mode entangled coherent state in a cavity.

In this paper, we propose a scheme to prepare two modes of a superconducting cavity in the multidimensional entangled coherent states of the form

$$\left|\Psi_N\right\rangle = \sum_{j=0}^{N-1} C_j \left|\alpha e^{-ij2\pi/N}\right\rangle_a \left|\alpha e^{ij2\pi/N}\right\rangle_b \tag{1}$$

which can be considered as the multi-dimensional generalization of entangled coherent state [12]. Recently, it is shown that such superposition states have larger amount of entanglement than entangled coherent states and find applications in quantum information processing [13]. Furthermore, we also point out that under certain condition, such superposition states can approximate pair coherent states [14] and superposition of pair coherent states [15] with a high degree of accuracy. Pair coherent states are regarded as an important type of correlated two-mode states, which can exhibit various nonclassical properties [14]. In reference [15], authors showed that pair cat states are characterized by additional nonclassical features beyond those of the pair coherent state. The experimental realization of these nonclassical states is of practical importance. In references [16,17], schemes have been proposed for generation of motional pair coherent state and pair cat states in a two-dimensional ion trap. In reference [18], Solano et al. propose a scheme for entangled coherent states in trapped ions. No schemes are proposed for generation of pair coherent states and superposition of pair coherent states in microwave cavity QED.

In order to present the principle idea of our scheme to generate entangled states (1), we rewritten equation (1) as follows

$$\left|\Psi_N\right\rangle = \sum_{j=0}^{N-1} C_j \exp\left[-\frac{ij2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right] \left|\alpha\right\rangle_a \left|\alpha\right\rangle_b. \tag{2}$$

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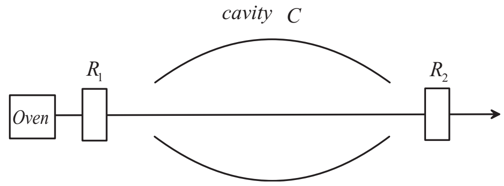

Fig. 1. Experimental apparatus. A sequence of atoms cross the cavity with same velocity. Outside the cavity, atoms are manipulated by classical fields R_1 and R_2, respectively.

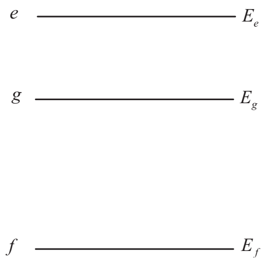

Fig. 2. Atomic level scheme with the corresponding energies.

It is easy to see that the polynomial on the righthand side of equation (2)can be factorized into a product of N terms

$$|\Psi_N\rangle = \prod_{m=1}^{N-1} \Bigg(\cos\theta_m - e^{i\varphi_m} \sin\theta_m \times \exp\left[-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right]\Bigg) |\alpha\rangle_a |\alpha\rangle_b \quad (3)$$

where parameters $x_j = e^{i\varphi_m}\tan\theta_m$ are complex roots of the equation $\sum_{j=0}^{N-1} C_j x^j = 0$. The factorization (3) suggests that we can prepare the state (1) from the coherent states $|\alpha\rangle_a|\alpha\rangle_b$ by applying $N-1$ times the transformation

$$|\Psi_m\rangle = \Bigg(\cos\theta_m - e^{i\varphi_m} \sin\theta_m \times \exp\left[-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right]\Bigg) |\Psi_{m-1}\rangle. \quad (4)$$

In the following, we will show that the transformation (4) can be implemented probabilistically in cavity QED.

The experimental setup is shown in Figure 1. Circular Rydberg atoms cross, one at a time, the superconducting cavity C, which sustains two nondegenerate orthogonally polarized modes M_a and M_b with the frequencies ω_a and ω_b. We assume that the atoms have energy-level configuration as that given in Figure 2. The $|e\rangle \leftrightarrow |g\rangle$ and $|f\rangle \leftrightarrow |g\rangle$ transitions are at 51.1 and 54.3 GHz, respectively. The cavity modes are shifted in the frequency from the transitions $|e\rangle \leftrightarrow |g\rangle$ and $|f\rangle \leftrightarrow |g\rangle$ by detunings δ_i and $\delta_{gf}^i = \delta_i + \delta_{det}$ $(i = a, b)$. The value $\delta_{det} = 3.2$ GHz is the frequency difference of the transitions $|e\rangle \leftrightarrow |g\rangle$ and $|f\rangle \leftrightarrow |g\rangle$. The experimental values, which are given in reference [9] show $\delta_i \ll \delta_{gf}^i$. Thus, we can choose the cavity frequencies in a way that only the levels $|e\rangle$ and $|g\rangle$ are appropriately affected by the nonresonant atom-field coupling. The quantum state $|f\rangle$ will in a good approximation not be affected during the atom-cavity interaction. Outside the cavity, the classical microwave field in Ramsey zones R_1 and R_2 induce resonant transition between state $|g\rangle$ and $|f\rangle$. Inside the cavity, the interaction Hamiltonian for the system in the interaction picture is given by

$$H = \Omega\Big(a|e\rangle\langle g|e^{i\delta_a t} + b|e\rangle\langle g|e^{i\delta_b t} + a^\dagger|g\rangle\langle e|e^{-i\delta_a t} + b^\dagger|g\rangle\langle e|e^{-i\delta_b t}\Big) \quad (5)$$

where a and $a^\dagger$ (b and $b^\dagger$) are annihilation and creation operators of mode M_a (M_b). $\delta_i = \omega_i - \omega_0$ is detuning between the frequency of the atomic transition and mode M_i $(i = a, b)$, where ω_0 is frequency of atomic transition $|e\rangle \leftrightarrow |g\rangle$. We assume that the $|e\rangle \leftrightarrow |g\rangle$ transition is coupled in the same way to both modes, with same vacuum Rabi oscillation frequency Ω. In the experiment [9], the atomic transition frequency ω_0 is tuned to the resonance with one mode, the interaction with the second one has a dispersive effect, provided that detuning is much larger than the vacuum Rabi oscillation frequency Ω. In this paper, we choose the atomic transition frequency ω_0 to be in the middle between the two modes frequencies $\omega_a - \omega_0 = \omega_0 - \omega_b$, i.e. $\delta_a = -\delta_b = \delta$. If the detuning δ is much bigger than the Rabi oscillation frequency Ω, it is convenient to consider the interaction (2) in terms of a coarse-grained Hamiltonian which neglects the effect of rapidly oscillating terms. Using the time-averaging method of reference [19], one can arrive at the effective Hamiltonian

$$H = \lambda(a^\dagger a - b^\dagger b)(|e\rangle\langle e| - |g\rangle\langle g|) \quad (6)$$

where $\lambda = \Omega^2/\delta$.

Now we consider the generation of entangled states (1) by sending a sequence of atoms through the cavity. For this purpose, we assume that the two cavity modes are initially prepared in the coherent states, $|\alpha\rangle_a|\alpha\rangle_b$, and the first atom is prepared in the superposition state $(|f_1\rangle + |e_1\rangle)/\sqrt{2}$ by a classical laser pulse in the Ramsey zone R_1. Thus as the atom enters the cavity, the state of the system is

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|f_1\rangle + |e_1\rangle\right)|\alpha\rangle_a|\alpha\rangle_b. \quad (7)$$

After passage through the cavity, the state of the system evolves into

$$\frac{1}{\sqrt{2}}\left[|f_1\rangle + |e_1\rangle \exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\alpha\rangle_a|\alpha\rangle_b \quad (8)$$

where the interaction time τ between atom and cavity is chosen to satisfy $\lambda\tau = 2\pi/N$. After leaving the cavity C, the atom is subjected to a classical pulse in the Ramsey zone R_2, which is tuned to the transition $|e_1\rangle \leftrightarrow |f_1\rangle$. The amplitude and the phase of the classical field is chosen appropriately so that the atom undergoes the transition

$$|e_1\rangle \longrightarrow (\cos\theta_1|e_1\rangle - \sin\theta_1 e^{i\varphi_1}|f_1\rangle)$$

$$|f_1\rangle \longrightarrow (\cos\theta_1|f_1\rangle + \sin\theta_1 e^{-i\varphi_1}|e_1\rangle)/\sqrt{2}$$

the parameters θ_1 and φ_1 are to be determined later. Thus, the state (8) becomes

$$\frac{1}{\sqrt{2}}\Big\{|f_1\rangle\left[\cos\theta_1 - \sin\theta_1 e^{i\varphi_1}\exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right] + |e_1\rangle\left[\cos\theta_1\exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right) + \sin\theta_1 e^{-i\varphi_1}\right]\Big\} \times |\alpha\rangle_a|\alpha\rangle_b. \quad (9)$$

If the atom is detected in state $|f_1\rangle$, the cavity fields are projected onto the state

$$|\psi_1\rangle = \frac{1}{\mathcal{N}_1}\left[\cos\theta_1 - \sin\theta e^{i\varphi_1} \times \exp\left(\frac{-i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\alpha\rangle_a|\alpha\rangle_b \quad (10)$$

where $\mathcal{N}_1$ is normalized factor. The state (10) is entangled coherent state [12], which has found application in quantum information processing [20]. The probability of finding the exiting atom in the expected states is $P_1 = |\mathcal{N}_1|^2/2$.

Next we send the second atom through the cavity which contains the field state (10) left by the first atom. In this process, the atom is still prepared in the superposition state $(|f_2\rangle + |e_2\rangle)/\sqrt{2}$. After the atom passing through the cavity with the duration τ, state of the system becomes

$$\frac{1}{\sqrt{2}}\left[|f_2\rangle + |e_2\rangle\exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\psi_1\rangle. \quad (11)$$

After atom exiting from the cavity, one detects whether the atom is in the state

$$\cos\theta_2|f_2\rangle - \sin\theta_2 e^{-i\varphi_2}|e_2\rangle. \quad (12)$$

This detection process can be implemented by passing the atom through the classical microwave field zone R_2 and field ionization counters, and the parameters θ_2 and φ_2 are to be determined later. If the atom is in the expected state, the state of the two cavity modes is projected into

$$|\Psi_2\rangle = \frac{1}{\mathcal{N}_2}\left[\cos\theta_2 - \sin\theta_2 e^{i\varphi_2} \times \exp\left(\frac{-i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\Psi_1\rangle \quad (13)$$

where $\mathcal{N}_2$ is normalized factor. The probability of finding the exiting atom in the expected states is $P_2 = |\mathcal{N}_2|^2/2$.

In general, after m such cycles of atomic excited state preparation, the atomic-cavity interaction, detection of atomic superposition of two states, the two cavity modes are prepared in the state $|\Psi_m\rangle$, which is multidimensional entangled coherent states. We now consider the $(m+1)$th cycle, i.e. the $(m+1)$th atom is initially prepared in the superposition state $(|f_{m+1}\rangle + |e_{m+1}\rangle)/\sqrt{2}$. After atom passing through the cavity with the interaction time τ, the quantum state becomes

$$\frac{1}{\sqrt{2}}\left(|f_{m+1}\rangle + |e_{m+1}\rangle\exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right)|\psi_m\rangle. \quad (14)$$

After atom exiting from the cavity, one detects whether the atom is in the state

$$\cos\theta_{m+1}|f_{m+1}\rangle - \sin\theta_{m+1}e^{-i\varphi_{m+1}}|e_{m+1}\rangle. \quad (15)$$

If the atom is in the expected state, the state of the two cavity modes is projected into

$$|\Psi_{m+1}\rangle = \frac{1}{\mathcal{N}_{m+1}}\left[\cos\theta_{m+1} - \sin\theta_{m+1}e^{i\varphi_{m+1}} \times \exp\left(\frac{-i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\Psi_m\rangle \quad (16)$$

where $\mathcal{N}_{m+1}$ is normalized factor. The probability of finding the exiting atom in the expected states is $P_{m+1} = |\mathcal{N}_{m+1}|^2/2$. Thus, after the procedure is performed for $N-1$ times the system's state definitely becomes

$$|\Psi_{N-1}\rangle = \frac{1}{\mathcal{N}_{N-1}}\left[\cos\theta_{N-1} - \sin\theta_{N-1}e^{i\varphi_N} \times \exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\Psi_{N-2}\rangle = \prod_{j=1}^{N-1}\frac{1}{\mathcal{N}_j}\left[\cos\theta_j - \sin\theta_j e^{i\varphi_j} \times \exp\left(-\frac{i2\pi}{N}\left(a^\dagger a - b^\dagger b\right)\right)\right]|\alpha\rangle_a|\alpha\rangle_b. \quad (17)$$

The success probability of the total scheme is $\prod_{i=1}^{N-1} P_i$. In order to generate the expected state (1), we choose the parameters $\tan\theta_1 e^{i\varphi_1}$, ..., $\tan\theta_{N-1}e^{i\varphi_{N-1}}$ to be the $N-1$ complex roots of the characteristic polynomial

$$\sum_{n=0}^{N-1} C_n(\tan\theta e^{i\varphi})^n = 0 \quad (18)$$

where C_n is determined by equation (1). In this case, equation (17) is proportional to state (1). This demonstrate the conditional generation of multidimensional entangled coherent states of two cavity modes. As an illustration, we consider the generation of entangled coherent states $\sum_{j=0}^{N-1}|\alpha e^{-ij2\pi/N}\rangle|\alpha e^{ij2\pi/N}\rangle$. For this purpose, we choose the parameters $\tan\theta_j = 1$ and $\varphi_j = 2j\pi/(N)$ $(j = 1, ..., N-1)$. The corresponding success probability is $|\sum_{m,n=0}^{N-1}\exp\{-2|\alpha|^2[1-\cos(2\pi(m-n)/N)]\}|^2/4^{N-1}$, which decreases exponentially with increasing N.

One application of the states generated by the procedures outlined above would be the generation of pair coherent state and pair cat states. Pair coherent states are regarded as an important type of correlated two-mode states [14]. In Fock state representation, pair coherent state [14] is defined as

$$|\Phi_q(\xi)\rangle = \mathcal{N}_q(|\xi|)\sum_{n=0}^{\infty}\frac{\xi^n}{\sqrt{n!(n+q)!}}|n+q, n\rangle \quad (19)$$

where $\mathcal{N}_q(|\xi|) = [|\xi|^{-q} J_q(2|\xi|)]^{-1/2}$ is the normalized coefficient and J_q is the modified Bessel function of the first kind of order q.

If we choose the coefficients of state (1) to satisfy $C_j = 1$, the state (1) can be rewritten as follows

$$|\Psi_N\rangle = [\mathcal{N}_0(|\alpha^2|)]^{-1}|\Phi_0(\alpha^2)\rangle + \sum_{s=N,2N,...} \alpha^s[\mathcal{N}_s(|\alpha^2|)]^{-1} \times [|\Phi_s(\alpha^2)\rangle + |\Phi_{-s}(\alpha^2)\rangle] \quad (20)$$

which is superposition of pair coherent states $|\Phi_{\pm s}(\alpha^2)\rangle$, $(s = 0, N, 2N, ...)$. If we choose $|\alpha|$ and N to satisfy condition $[\mathcal{N}_0(|\alpha^2|)]^{-1} \gg |\alpha^s|[\mathcal{N}_{\pm s}(|\alpha^2|)]^{-1}$ $(s = N, 2N, ...)$, only the first term $|\Phi_0(\alpha^2)\rangle$ is important and pair coherent state is approximately generated. One can qualify how close the state (18) is to the pair coherent state in terms of the fidelity $F = |\langle\Phi_0(\alpha^2)|\Psi_N\rangle|^2$. If $\alpha = 2.5$ and $N = 15$, the cavity fields are prepared in the pair coherent state with a fidelity higher than 0.99.

If we choose the coefficients of state (1) to satisfy $C_j = \cos(2j\pi/N)$, the state (1) can be rewritten as follows

$$\begin{aligned}|\Psi_N\rangle &= [\mathcal{N}_1(|\alpha^2|)]^{-1}[|\Phi_1(\alpha^2)\rangle + |\Phi_{-1}(\alpha^2)\rangle] \\ &+ \sum_{s=N,2N,...} \alpha^s[\mathcal{N}_{s+1}(|\alpha^2|)]^{-1}[|\Phi_{s+1}(\alpha^2)\rangle + |\Phi_{-s-1}(\alpha^2)\rangle] \\ &+ \sum_{s=N,2N,...} \alpha^{s-2}[\mathcal{N}_{s-1}(|\alpha^2|)]^{-1}[|\Phi_{s-1}(\alpha^2)\rangle + |\Phi_{-s+1}(\alpha^2)\rangle]\end{aligned} \quad (21)$$

when $|\alpha|$ and N are chosen to satisfy condition $[\mathcal{N}_1(|\alpha^2|)]^{-1} \gg |\alpha^s|[\mathcal{N}_{s+1}(|\alpha^2|)]^{-1}$ and $|\alpha^{s-2}|[\mathcal{N}_{s-1}(|\alpha^2|)]^{-1}$ $(s = N, 2N, ...)$, only the first term of equation (21) is important. In this case pair cat state is approximately generated.

In summary, we proposed a scheme to generate the multidimensional entangled coherent states of two cavity modes. This scheme of quantum state generation includes the generation of entangled coherent states. It is also shown that under certain condition such superposition state can approximate pair coherent state and pair cat state with a high degree of accuracy. This schemes provides a new way for engineering quantum entanglement between two cavity modes via discrete superposition of two-mode coherent states. This scheme can be adapted in a straightforward way to generate entangled states in spatially separated cavity modes.

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